Corrections to the apparent value of the cosmological constant due to local inhomogeneities

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Abstract

Supernovae observations strongly support the presence of a cosmological constant, but its value, which we will call apparent, is normally determined assuming that the Universe can be accurately described by a homogeneous model. Even in the presence of a cosmological constant we cannot exclude nevertheless the presence of a small local inhomogeneity which could affect the apparent value of the cosmological constant. Neglecting the presence of the inhomogeneity can in fact introduce a systematic misinterpretation of cosmological data, leading to the distinction between an apparent and true value of the cosmological constant. We establish the theoretical framework to calculate the corrections to the apparent value of the cosmological constant by modeling the local inhomogeneity with a ΛLTB solution. Our assumption to be at the center of a spherically symmetric inhomogeneous matter distribution correspond to effectively calculate the monopole contribution of the large scale inhomogeneities surrounding us, which we expect to be the dominant one, because of other observations supporting a high level of isotropy of the Universe around us.

By performing a local Taylor expansion we analyze the number of independent degrees of freedom which determine the local shape of the inhomogeneity, and consider the issue of central smoothness, showing how the same correction can correspond to different inhomogeneity profiles. Contrary to previous attempts to fit data using large void models our approach is quite general. The correction to the apparent value of the cosmological constant is in fact present for local inhomogeneities of any size, and should always be taken appropriately into account both theoretically and observationally.

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I. INTRODUCTION

High redshift luminosity distance measurements [1–6] and the WMAP measurements [7, 8] of cosmic microwave background (CMB) interpreted in the context of standard FLRW cosmological models have strongly disfavored a matter dominated universe, and strongly supported a dominant dark energy component, giving rise to a positive cosmological acceleration.

As an alternative to dark energy, it has been proposed [9, 10] that we may be at the center of an inhomogeneous isotropic universe without cosmological constant described by a Lemaitre-Tolman-Bondi (LTB) solution of Einstein's field equations, where spatial averaging over one expanding and one contracting region is producing a positive averaged acceleration a_D, but it has been shown how spatial averaging can give rise to averaged quantities which are not observable [11]. Another more general approach to map luminosity distance as a function of redshift $D_L(z)$ to LTB models has been recently proposed [12, 13], showing that an inversion method can be applied successfully to reproduce the observed $D_L(z)$. Interesting analysis of observational data in inhomogeneous models without dark energy and of other theoretically related problems is given for example in [14–31]

Here in this paper we will adopt a different approach. We will consider a Universe with a cosmological constant and some local large scale inhomogeneity modeled by a ΛLTB solution [32]. For simplicity we will also assume that we are located at its center. In this regard this can be considered a first attempt to model local large scale inhomogeneities in the presence of the cosmological constant or, more in general, dark energy. Given the spherical symmetry of the LTB solution and the assumption to be located at the center our calculation can be interpreted as the monopole contribution of the large inhomogeneities which surround us. Since we know from other observations such as CMB radiation that the Universe appears to be highly isotropic, we can safely assume that the monopole contribution we calculate should also be the dominant one, making our results even more relevant. After calculating the null radial geodesics for a central observer we then compute the luminosity distance and compare it to that of ΛCDM model, finding the relation between the two different cosmological constants appearing in the two models, where we call apparent the one in the ΛCDM and true the one in ΛLTB. Our calculations show that the corrections to Ω_Λ^{app}, which is the value of the cosmological constant obtained from analyzing supernovae data assuming homogeneity, can be important and should be taken into account.

II. LTB SOLUTION WITH A COSMOLOGICAL CONSTANT

The LTB solution can be written as [33–35] as

$$ds^2 = -dt^2 + \frac{(R_{,r})^2\, dr^2}{1 + 2\, E(r)} + R^2 d\Omega^2 \,, \tag{1}$$

where R is a function of the time coordinate t and the radial coordinate r, $E(r)$ is an arbitrary function of r, and $R_{,r} = \partial_r R(t, r)$. The Einstein equations with dust and a cosmological constant give

$$\left(\frac{\dot{R}}{R}\right)^2 = \frac{2E(r)}{R^2} + \frac{2M(r)}{R^3} + \frac{\Lambda}{3} \,, \tag{2}$$

$$\rho(t, r) = \frac{2M_{,r}}{R^2 R_{,r}} \,, \tag{3}$$

with $M(r)$ being an arbitrary function of r, $\dot{R} = \partial_t R(t, r)$ and $c = 8\pi G = 1$ is assumed throughout the paper. Since Eq. (2) contains partial derivatives respect to time only, its general solution can be obtained from the FLRW equivalent solution by making every constant in the latter one an arbitrary function of r.

The general analytical solution for a FLRW model with dust and cosmological constant was obtained by Edwards [36] in terms of elliptic functions. By an appropriate choice of variables and coordinates, we may extend it to the LTB case thanks to the spherical symmetry of both LTB and FLRW models, and to the fact that dust follows geodesics without being affected by adjacent regions. An anaytical solution can be found by introducing a new coordinate $\eta = \eta(t, r)$ and a variable a by

$$\left(\frac{\partial \eta}{\partial t}\right)_r = \frac{r}{R} \equiv \frac{1}{a} \,, \tag{4}$$

and new functions by

$$\rho_0(r) \equiv \frac{6M(r)}{r^3} \,, \quad k(r) \equiv -\frac{2E(r)}{r^2} \,. \tag{5}$$

Then Eq. (2) becomes

$$\left(\frac{\partial a}{\partial \eta}\right)^2 = -k(r) a^2 + \frac{\rho_0(r)}{3} a + \frac{\Lambda}{3} a^4 \,, \tag{6}$$

where a is now regarded as a function of η and r, $a = a(\eta, r)$. It should be noted that the coordinate η, which is a generalization of the conformal time in a homogeneous FLRW universe, has been only implicitly defined by Eq. (4). The actual relation between t and η can be obtained by integration once $a(\eta, r)$ is known:

$$t(\eta, r) = \int_0^\eta a(x, r) dx + t_b(r) \,, \tag{7}$$

which can be computed analytically, and involve elliptic integrals of the third kind[37].

The function $t_B(r)$ plays the role of constant of integration, and is an arbitrary function of r, sometime called bang function, since by construction at time $t = t_b(r)$ we have $a(t_b(r), r) = 0$, and correspond to the fact that the big bang initial singularity can happen at different times at different positions from the center in a LTB space. In the rest of this paper we will assume homogeneous bang, i.e. we will set

$$t_b(r) = 0. \tag{8}$$

Inspired by the construction of the solution for the FLRW case get:

$$a(\eta, r) = \frac{\rho_0(r)}{3\phi\left(\frac{\eta}{2}; g_2(r), g_3(r)\right) + k(r)}, \tag{9}$$

where $\phi(x; g_2, g_3)$ is the Weierstrass elliptic function satisfying the differential equation

$$\left(\frac{d\phi}{dx}\right)^2 = 4\phi^3 - g_2\phi - g_3\,, \tag{10}$$

and

$$\alpha = \rho_0(r)\,, \quad g_2 = \frac{4}{3}k(r)^2\,, \quad g_3 = \frac{4}{27}\left(2k(r)^3 - \Lambda\rho_0(r)^2\right)\,. \tag{11}$$

In this paper we will choose the so called FLRW gauge, i.e. the coordinate system in which $\rho_0(r)$ is constant.

III. GEODESIC EQUATIONS AND LUMINOSITY DISTANCE

We adopt the same method developed in [38] to solve the null geodesic equation written in terms of the coordinates (η, r). Instead of integrating differential equations numerically, we perform a local expansion of the solution around $z = 0$ corresponding to the point $(t_0, 0)$, or equivalently $(\eta_0, 0)$, where $t_0 = t(\eta_0, 0)$. The change of variables from (t, r) to (η, r) permits us to have r.h.s. of all equations in a fully analytical form, in contrast to previous considerations of this problem which require a numerical calculation of $R(t, r)$ from the Einstein equation (2). Thus, this formulation is particularly suitable for derivation of analytical results.

The luminosity distance for a central observer in the LTB space-time as a function of the redshift z is expressed as

$$D_L(z) = (1+z)^2 R\left(t(z), r(z)\right) = (1+z)^2 r(z) a\left(\eta(z), r(z)\right)\,, \tag{12}$$

where $\Big(t(z), r(z)\Big)$ or $\Big((\eta(z), r(z)\Big)$ is the solution of the radial geodesic equation as a function of z. The past-directed radial null geodesics is given by

$$\frac{dt}{dr} = -\frac{R_{,r}(t,r)}{\sqrt{1+2E(r)}} \,. \tag{13}$$

In terms of z, Eq. (13) takes the form [39]:

$$\begin{aligned} \frac{dr}{dz} &= \frac{\sqrt{1+2E(r(z))}}{(1+z)\dot{R}_{,r}[r(z),t(z)]} \,, \\ \frac{dt}{dz} &= -\frac{R_{,r}[r(z),t(z)]}{(1+z)\dot{R}_{,r}[r(z),t(z)]} \,. \end{aligned} \tag{14}$$

The inconvenience of using the (t, r) coordinates is that there is no exact analytical solution for $R(t, r)$. So the r.h.s. of Eqs. (14) cannot be evaluated analytically, but we are required to find a numerical solution for R first [40], and then to integrate numerically the differential equations, which is quite an inconvenient and cumbersome procedure, and cannot be used to derive anaytical results.

It can be shown [38] that in the coordinates (η, r) eqs. (14) take the form:

$$\frac{d\eta}{dz} = -\frac{\partial_r t(\eta,r) + F(\eta,r)}{(1+z)\partial_\eta F(\eta,r)} \equiv p(\eta,r) \,, \tag{15}$$

$$\frac{dr}{dz} = \frac{a(\eta,r)}{(1+z)\partial_\eta F(\eta,r)} \equiv q(\eta,r) \,, \tag{16}$$

where

$$F(\eta,r) \equiv \frac{R_{,r}}{\sqrt{1+2E(r)}} = \frac{1}{\sqrt{1-k(r)r^2}}\Big[\partial_r(a(\eta,r)r) - a^{-1}\partial_\eta(a(\eta,r)r)\,\partial_r t(\eta,r)\Big] \,. \tag{17}$$

It is important to observe that the functions p, q, F have explicit analytical forms, making it particularly useful do derive anaytical results.

IV. NUMBER OF INDEPENDENT PARAMETERS AND TAYLOR EXPANSION ACCURACY

In order to find the relation between the apparent and true value of the cosmological constant we need in to match the terms in the red-shift expansion :

$$D_i^{\Lambda CDM} = D_i^{\Lambda LTB} \,, \tag{18}$$

Before proceeding in deriving this relation we need to understand clearly how many independent parameters we can solve for at different order in the Taylor expansion for $D_L(z)$. After defining the expansion of the function $k(r)$ in terms of the dimensionless function $K(r)$:

$$k(r) = (a_0 H_0)^2 K(r) = K_0 + K_1 r + K_2 r^2 + .. \tag{19}$$

we have

$$D_1^{\Lambda LTB} = \frac{1}{H_0^{\Lambda LTB}}, \tag{20}$$

$$D_i^{\Lambda LTB} = f_i(\Omega_\Lambda, K_0, K_1, .., K_{i-1}), \tag{21}$$

which implies that if we want to match the coefficient D_i up to order n, we will have a total of $n+2$ independent parameters to solve for :

$$\{H_0^\Lambda, \Omega_\Lambda, K_0, K_i, .., K_{i-1}\}. \tag{22}$$

The matching conditions will imply a constraint over the $n+2$ independent parameters, but this will not be enough completely determine them, since two of them will always be free. For a matter of computational convenience we will choose K_0, K_1 as free parameters and express all the other in terms of them. For example from :

$$D_2^{\Lambda CDM} = D_2^{\Lambda LTB}, \tag{23}$$

we can get

$$\Omega_\Lambda^{app}(\Omega_\Lambda, K_0, K_1), \tag{24}$$

from

$$D_3^{\Lambda CDM} = D_3^{\Lambda LTB}, \tag{25}$$

we can get

$$K_2(\Omega_\Lambda^{app}, K_0, K_1), \tag{26}$$

and in general from

$$D_i^{\Lambda CDM} = D_i^{\Lambda LTB}, \tag{27}$$

we can get

$$K_{i-1}(\Omega_\Lambda^{app}, K_0, K_1). \tag{28}$$

Since our purpose is to find the corrections to the apparent value of the cosmological constant, the second order term D_2 is enough. Higher order terms in the redshift expansion will provide

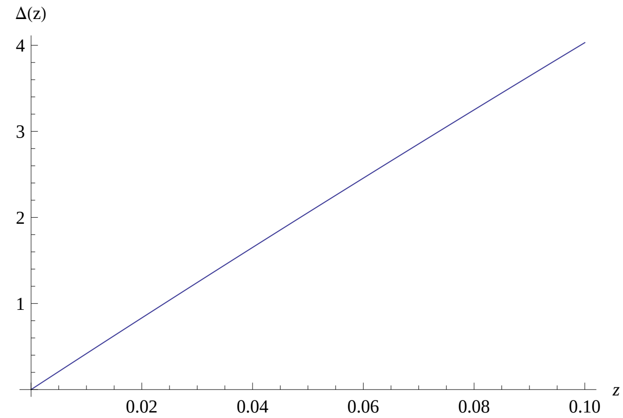

FIG. 1: The percentual error $\Delta = 100\frac{D^{\Lambda CDM}-D^{\Lambda CDM}_{Taylor}}{D^{\Lambda CDM}}$ for a third order expansion is plotted as a function of the redshift. As it can be seen the error is already quite large at redshift 0.1. Higher order expansion does not improve the convergence.

$K_2, K_3, ..K_{i-1}$ as functions of $\{\Omega_\Lambda^{app}, K_0, K_1\}$, but will not change the analytical relation between Ω_Λ^{app} and Ω_Λ which can be derived from eq.(23). For this reason we will only need the expansion up to second order for the luminosity distance. The fact that we have more free parameters than constraints implies that the same correction to the apparent value of the cosmological constant can correspond to an infinite number of different inhomogeneity profiles.

The corrections we calculate are accurate within the limits of validity of the Taylor expansion $D^{\Lambda CDM}_{Taylor}$. It turns out that in the flat case we consider the error is quite large already at a redshift of about 0.2 as shown in the figure. This implies that the corrections should also be valid only within this low redshift range, since even if we are exactly matching the coefficients, the Taylor expansion of the ΛCDM best fit formula itself is not very accurate. This could be overcome by implementing other types of expansions or numerical methods, such as Padé for example, with better convergence behavior, but we'll leave this to a future work.

V. CENTRAL BEHAVIOR

A function of the radial coordinate $f(r)$ is smooth at the center $r = 0$ only if all its odd derivatives vanish there. This can be shown easily by looking at the partial derivatives of even order of this type for example:

$$\partial_x^{2n}\partial_y^{2n}\partial_z^{2n} f(\sqrt{x^2+y^2+z^2})\,, \tag{29}$$

where $\{x, y, z\}$ are the cartesian coordinates related to r by $r^2 = x^2 + y^2 + z^2$. Quantities of the type above diverge at the center if $\partial_r^{2m+1} f(r) \neq 0$ for $2m + 1 < 2n$. If for example the first derivative $f'(0)$ is not zero, then the laplacian will diverge. This implies that including linear terms expansions for $k(r)$ and $t_b(r)$ we are considering models which are not smooth at the center. The general central smoothness conditions are:

$$k_{2m+1} = 0, \tag{30}$$

$$t_b^{2m+1} = 0\,, \tag{31}$$

$$2m + 1 < i\,, \tag{32}$$

which must be satisfied for all the relevant odd powers coefficients of the central Taylor expansion. In our case this implies that if we only want to consider centrally smooth inhomogeneities then we need to set to zero all the odd derivatives of $K(r)$

$$K_{2m+1} = 0 \tag{33}$$

The consequence of this smoothness conditions is that the exact matching of the Taylor expansion is possible only up to order five when we have five constraints equations

$$D_i^{\Lambda CDM} = D_i^{\Lambda LTB} \quad , \quad 1 \leq i \leq 5\,, \tag{34}$$

and five free parameters

$$H_0^{\Lambda LTB}, \Omega_\Lambda, K_0, K_2, K_4 \tag{35}$$

implying there is a unique solution. Going to higher order there will be more equations than free parameters making the inversion problem impossible. This means that the effects of a different value of the cosmological constant cannot be mimicked by a smooth inhomogeneity, as far as the exact matching of the Taylor expansion is concerned. From a data analysis point of view this limitation could be easily circumvented, since these considerations are based on matching the Taylor expansion of the best ΛCDM fit, which is quite different from fitting the actual data. Also it turns out that the Taylor expansion $D_{Taylor}^{\Lambda CDM}(z)$ is more accurate at second order than at any other order as shown in the figure, implying that exact matching beyond second order is practically irrelevant from a data fitting point of view. Under these considerations the inversion problem can be considered still effectively undetermined since by matching up to second order we have two equations and three parameters:

$$H_0^{\Lambda LTB}, \Omega_\Lambda, K_0 \tag{36}$$

For completeness of the analysis we mention that after counting the number of independent parameters we can easily conclude that the inversion problem remain undetermined for the third order, and has a unique solution for the fourth and fifth order as shown above.

VI. CALCULATING THE LUMINOSITY DISTANCE

In order to obtain the redshift expansion of the luminosity distance we need to use the following:

$$k(r) = (a_0 H_0)^2 K(r) = K_0 + K_1 r + K_2 r^2 + .. \tag{37}$$

$$t(\eta, r) = b_0(\eta) + b_1(\eta) r + b_2(\eta) r^2 + .. \tag{38}$$

It should be noted that linear terms will in fact lead to central divergences of the laplacian in spherical coordinates, which correspond to a central spike of the energy distribution [29, 30], but an appropriate local averaging of the solution can easily heal this behavior, and we include them here because they give the leading order contribution. Since we are interested in the effects due to the inhomogeneities we will neglect k_0 in the rest of the calculation because this corresponds to the homogeneous component of the curvature function $k(r)$.

Following the same approach given in [32] , we can find a local Taylor expansion in red-shift for the geodesics equations, and then calculate the luminosity distance:

$$D_L^{\Lambda LTB}(z) = (1+z)^2 r(z) a^{\Lambda LTB}(\eta(z), r(z)) = D_1^{\Lambda LTB} z + D_2^{\Lambda LTB} z^2 + D_3^{\Lambda LTB} z^3 + .. \tag{39}$$

$$D_1^{\Lambda LTB} = \frac{1}{H_0},$$

$$\begin{aligned} D_2^{\Lambda LTB} = {} & \frac{1}{36 H_0 (\Omega_\Lambda^{true} - 1)} \Big[54 B_1 (\Omega_\Lambda^{true} - 1)^2 + 18 B_1' (\Omega_\Lambda^{true} - 1) - 18 h_{0,r} (\Omega_\Lambda^{true})^2 \\ & + 30 h_{0,r} \Omega_\Lambda^{true} - 12 h_{0,r} + 6 K_1 \Omega_\Lambda^{true} - 10 K_1 + 27 (\Omega_\Lambda^{true})^2 - 18 \Omega_\Lambda^{true} - 9 \Big], \end{aligned} \tag{40}$$

where we have introduced the dimensionless quantities $K_0, K_1, B_1, B_1', h_{0,r}$ according to

$$H_0 = \left(\frac{\partial_t, a(t,r)}{a(t,r)} \right)^2 \Bigg|_{t=t_0, r=0} = \left(\frac{\partial_\eta a(\eta,r)}{a(\eta,r)^2} \right)^2 \Bigg|_{\eta=\eta_0, r=0}, \tag{41}$$

$$B_1(\eta) = b_1(\eta) a_0^{-1}, \tag{42}$$

$$B_1 = b_1(\eta_0) a_0^{-1}, \tag{43}$$

$$B_1' = \frac{\partial B_1(\eta)}{\partial \eta} \Bigg|_{\eta=\eta_0} (a_0 H_0)^{-2}, \tag{44}$$

$$h_{0,r} = \frac{1}{a_0 H_0} \frac{\partial_r a(\eta, r)}{a(\eta, r)} \Bigg|_{\eta=\eta_0, r=0}, \tag{45}$$

$$t_0 = t(\eta_0, 0), \tag{46}$$

and used the Einstein equation at the center ($\eta = \eta_0, r = 0$)

$$\begin{aligned} 1 &= \Omega_k(0) + \Omega_M + \Omega_\Lambda = -K_0 + \Omega_M + \Omega_\Lambda, & (47) \\ \Omega_k(r) &= -\frac{k(r)}{H_0^2 a_0^2}, & (48) \\ \Omega_M &= \frac{\rho_0}{3H_0^2 a_0^3}, & (49) \\ \Omega_\Lambda &= \frac{\Lambda}{3H_0^2}. & (50) \end{aligned}$$

Because of our coordinate choice Ω_M is independent of r, and all the radial dependence goes into $\Omega_k(r)$. Note that apart from the central curvature term K_0, the inhomogeneity of the LTB space is expressed in $h_{0,r}$, which encodes the radial dependence of the scale factor. Details of these rather cumbersome calculations are provided in a separate companion paper, but it should be emphasized that in order to put the formula for the luminosity distance in this form it is necessary to manipulate appropriately the elliptic functions and then re-express everything in terms of physically meaningful quantities such as H_0.

VII. CALCULATING $D_L(z)$ FOR ΛCDM MODELS.

The metric of a ΛCDM model is the FLRW metric, a special case of LTB solution, where :

$$\begin{aligned} \rho_0(r) &\propto const, & (51) \\ k(r) &= 0, & (52) \\ t_b(r) &= 0, & (53) \\ a(t,r) &= a(t). & (54) \end{aligned}$$

We will calculate independently the expansion of the luminosity distance and the redshift spherical shell mass for the case of a flat ΛCDM, to clearly show the meaning of our notation, and in particular the distinction between Ω_Λ^{app} and Ω_Λ^{true}. We can also use these formulas to check the results derive before, since in absence of inhomogeneities they should coincide.

One of the Einstein equation can be expressed as:

$$H^{\Lambda CDM}(z) = H_0\sqrt{(1-\Omega_\Lambda^{app})\left(\frac{a_0}{a}\right)^3 + \Omega_\Lambda^{app}} = H_0\sqrt{(1-\Omega_\Lambda^{app})(1+z)^3 + \Omega_\Lambda^{app}}. \quad (55)$$

We can then calculate the luminosity distance using the following relation, which is only valid assuming flatness:

$$D_L^{\Lambda CDM}(z) = (1+z)\int_0^z \frac{dz'}{H^{\Lambda CDM}(z')} = D_1^{\Lambda CDM} z + D_2^{\Lambda CDM} z^2 + D_3^{\Lambda CDM} z^3 + \dots \quad (56)$$

From which we can get:

$$D_1^{\Lambda CDM} = \frac{1}{H_0}, \tag{57}$$

$$D_2^{\Lambda CDM} = \frac{3\Omega_\Lambda^{app}+1}{4H_0}. \tag{58}$$

We can check the consistency between these formulae and the ones derived in the case of LTB by setting:

$$K_1 = B_1 = B_1' = K_0 = h_{0,r} = 0, \tag{59}$$

$$\tag{60}$$

which corresponds to the case in which $\Omega_\Lambda^{app} = \Omega_\Lambda^{true}$.

VIII. RELATION BETWEEN APPARENT AND TRUE VALUE OF THE COSMOLOGICAL CONSTANT

So far we have calculated the first two terms of the redshift expansion of the luminosity distance for ΛLTB and ΛCDM model. Since we now that the latter provides a good fitting for supernovae observations, we can now look for the ΛLTB models which give the same theoretical prediction. From the above relations we can derive :

$$H_0^{\Lambda LTB} = H_0^{\Lambda CDM}, \tag{61}$$

$$\begin{aligned}\Omega_\Lambda^{app} = \frac{1}{27(\Omega_\Lambda^{true}-1)}\Big[&54B_1(\Omega_\Lambda^{true})^2 - 108B_1\Omega_\Lambda^{true} + 54B_1 + 18B_1'\Omega_\Lambda^{true} - 18B_1'\\ &-18h_{0,r}(\Omega_\Lambda^{true})^2 + 30h_{0,r}\Omega_\Lambda^{true} - 12h_{0,r} + 6K_1\Omega_\Lambda^{true} - 10K_1\\ &+27\Omega_\Lambda^{true}(\Omega_\Lambda^{true}-1)\Big],\end{aligned} \tag{62}$$

$$\begin{aligned}\Omega_\Lambda^{true} = -\frac{1}{6(6B_1-2h_{0,r}+3)}\Bigg[&\Big((36B_1 - 6B_1' - 10h_{0,r} - 2K_1 + 9\Omega_\Lambda^{app} + 9)^2 +\\ &-4(6B_1-2h_{0,r}+3)(54B_1 - 18B_1' - 12h_{0,r} - 10K_1 + 27\Omega_\Lambda^{app})\Big)^{1/2} - 36B_1\\ &+6B_1' + 10h_{0,r} + 2K_1 - 9(\Omega_\Lambda^{app}-1)\Bigg].\end{aligned} \tag{63}$$

We can also expand the above exact relations assuming that all the inhomogeneities, can be treated perturbatively respect to the ΛCDM , i.e. $\{K_1, B_1, B_1'\} \propto \epsilon$, where ϵ stands for a small deviation from $FLRW$ solution :

$$\begin{aligned}\Omega_\Lambda^{true} = \Omega_\Lambda^{app} - \frac{2}{27(\Omega_\Lambda^{app}-1)}&(27B_1(\Omega_\Lambda^{app}-1)^2 + 9B_1'(\Omega_\Lambda^{app}-1) - 9h_{0,r}(\Omega_\Lambda^{app})^2 + 15h_{0,r}\Omega_\Lambda^{app}\\ &-6h_{0,r} + 3K_1\Omega_\Lambda^{app} - 5K_1) + O(\epsilon^2).\end{aligned} \tag{64}$$

As expected all these relations reduce to

$$\Omega_\Lambda^{true} = \Omega_\Lambda^{app}, \tag{65}$$

in the limit in which there is no inhomogeneity, i.e. when $K_1 = B_1 = B_1' = h_{0,r} = 0$.

IX. CONCLUSIONS

We have derived for the first time the correction due to local large scale inhomogeneities to the value of the apparent cosmological constant inferred from low redshift supernovae observations. This analytical calculation shows how the presence of a local inhomogeneity can affect the estimation of the value of cosmological parameters, such as Ω_Λ. This effects should be properly taken into account both theoretically and observationally. By performing a local Taylor expansion we analyzed the number of independent degrees of freedom which determine the local shape of the inhomogeneity, and consider the issue of central smoothness, showing how the same correction can correspond to different inhomogeneity profiles. We will address in a future work the estimation of the magnitude of this effect based on experimental bounds which can be set on the size and shape of a local inhomogeneity and the fitting of actual supernovae data. It is important to underline here that we do not need a large void as normally assumed in previous studies of LTB models in a cosmological context. Even a small inhomogeneity could in fact be important.

In the future it will also be interesting to extend the same analysis to other observables such as barionic acoustic oscillations (BAO) or the cosmic microwave background radiation (CMBR), and we will report about this in separate papers. Another direction in which the present work could be extended is modeling the local inhomogeneity in a more general way, for example considering not spherically symmetric solutions. From this point of view our calculation could be considered the monopole contribution to the general effect due to a local large scale inhomogeneity of arbitrary shape. Given the high level of isotropy of the Universe shown by other observations such as the CMB radiation, we can expect the monopole contribution we calculated to be the dominant one.

While this should be considered only as the first step towards a full inclusion of the effects of large scale inhomogeneities in the interpretation of cosmological observations, it is important to emphasize that we have introduced a general definition of the concept of apparent and true value of cosmological parameters, and shown the general theoretical approach to calculate the

corrections to the apparent values obtained under the standard assumption of homogeneity.

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